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SOME PROPERTIES OF THE PRESSURE–ENTROPY DIAGRAM

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Abstract—Some explanations are given upon properties of the pressure–entropy diagram for liquid and steam and for ideal gases.

The representation on this graph not only of classical thermodynamic cycles, but also of reheat and regenerative modern cycles, shows that it may be considered as a useful complement of Mollier diagrams.

NOMENCLATURE

p,	absolute pressure;
T,	absolute temperature;
v,	specific volume;
s,	specific entropy;
s_{mol},	molal entropy;
C,	molal specific heat;
n,	exponent of polytropic lines;
R,	universal gas constant;
k,	specific heat ratio.

1.

IN THE $[p, s]$ diagram the isobaric and isentropic lines, which represent common and important thermodynamic processes, are obviously straight lines, parallel to the co-ordinate axis.

So the Rankine–Clausius cycle is represented by the rectangle $ABCD$* in Fig. 1, in which the ordinates (between 10^{-3} and 10^{3} bar) are $\log p$, and the abscissae are the values of the adimensional group $[s_{mol}/R]$, which may be named number of Clausius.

The segments AB and BL are the compression and the heating lines of liquid, LV and VC the evaporation and superheating lines, CD and AD the expansion and the condensation of steam.

The field of liquid, that is actually of considerable importance, is greatly amplified.

The rectangle $AB_0\,C_0D$, between 0·03 and 300 bar, represents a cycle which is enlarged to the hypercritical field.

For thermodynamic and technological reasons this cycle has to be modified by the fractionated compression $ABLB_1L_1B_n$ and the fractionated expansion $C_n\ V_2\ C_2\ V_1C_1D$, being 600°C, for instance, the maximum attainable value of the temperature.

The regenerative heat processes take place in an intermediate field of temperatures, between CD and BL, C_1C and B_1L_1 and so on.

The thermal energy for heating the liquid is done at BL and B_1L_1 with a step-by-step method by means of condensation of portions of steam which expands between C_1 and C and respectively between C and D.

The segments $V_2\ C_2$ and $V_1\ C_1$ are reheat isobaric processes; $C_n\ V_2$ and $C_2\ V_1$ are respectively high pressure and middle pressure isentropic expansions.

2.

For a polytropic process of an ideal gas we have:

$$\frac{dp}{p} + \frac{dv}{v} = \frac{dT}{T} \quad \text{equation of state} \qquad (1)$$

$$\frac{dp}{p} + n\,\frac{dv}{v} = 0 \quad \text{equation of the process} \qquad (2)$$

$$ds_{mol} = C\,\frac{dT}{T} \quad \text{equation of entropy} \qquad (3)$$

from which; since $n = (C - C_p)/(C - C_v)$ and $R = C_p - C_v$, we derive:

$$\left[\frac{\partial(\ln p)}{\partial(s_{mol}/R)}\right]_C = \frac{C_p}{C} - 1 = \text{const.} \qquad (4)$$

All the polytropic processes of ideal gases are then represented by straight lines in $[\ln p,$

* Cfr. C. Codegone, Recenti sviluppi della Termodinamica applicata, *Riv. Nuovo Cimento* III, 2 (1947).

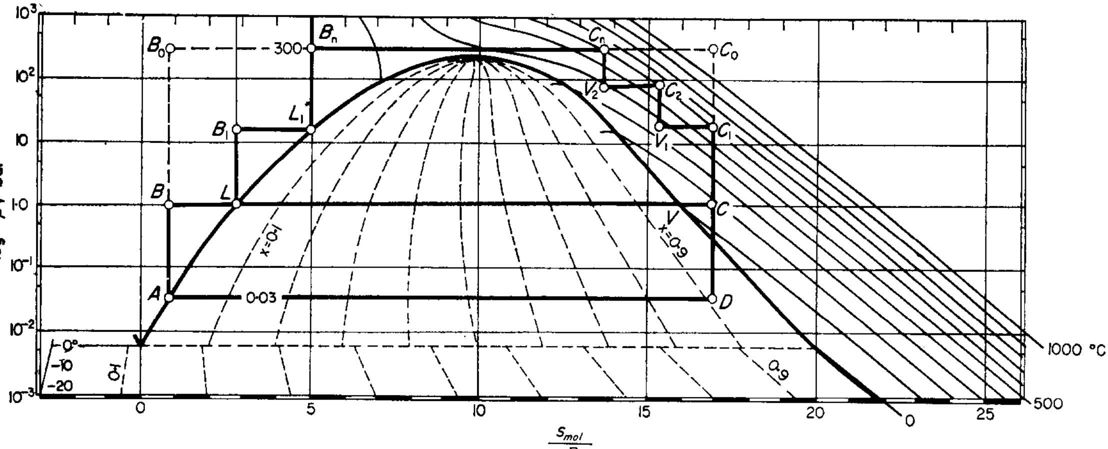

FIG. 1. p, s diagram for H_2O.

s_{mol}/R] diagram. In particular for isothermal and isenthalpic processes:

$$\left[\frac{\partial\,(\ln p)}{\partial\,(s_{mol}/R)}\right]_T = -\,1 \tag{5}$$

and for isometric processes:

$$\left[\frac{\partial\,(\ln p)}{\partial\,(s_{mol}/R)}\right]_v = k - 1. \tag{6}$$

Along an isentropic line the temperature and the volume follow the laws:

$$\left[\frac{\partial\,(\ln T)}{\partial\,(\ln p}\right]_s = \frac{k-1}{k}\,; \qquad \left[\frac{\partial\,(\ln v)}{\partial\,(\ln p)}\right]_s = -\frac{1}{k}. \tag{7}$$

The lengths of the logarithmic unities for p, T and v are therefore in the proportion 1: $(k/k-1)$: k.

Moreover, along an isobaric process:

$$\left[\frac{\partial\,(s_{mol}\,R)}{\partial\,(\ln T)}\right]_p = \frac{C_p}{R} = \frac{k}{k-1}. \tag{8}$$

With reference to common logarithms the lengths of the logarithmic unities for p and [$s_{mol}/2{\cdot}3\,R$] may be equal.

Fig. 2 gives a [p, s] diagram for diatomic ideal gases ($k = 1{\cdot}4$) in adimensional co-ordinates.

The rectangle $OJ_1\,J_2\,J_3$ represents a Joule cycle and the trapezium $OD_1\,D_2\,D_3$ a Diesel cycle, while parallelograms $OC_1\,C_2\,C_3$ and $OO_1\,O_2\,O_3$ represent a Carnot and an Otto cycle respectively.

The isothermal compression OC_1 and the isothermal expansion $C_2\,C_3$ of the Carnot cycle, which are difficult to actuate with gases, may be approximated by para-isothermal processes, i.e. by series of isentropic (OC_1'; $C_1''\,C_1'''$; etc.) and isobaric ($C_1'\,C_1''$; etc.) lines.

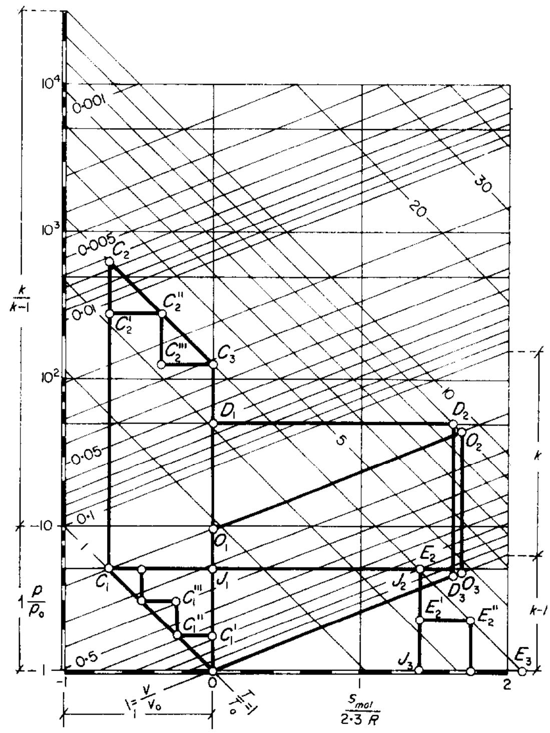

FIG. 2. p, s diagram for ideal gases ($k = 1{\cdot}4$).

The same method may be applied to the isothermal processes OC_1 and $E_2 E_3$ of the Ericson regenerative cycle of gas-turbine plants.

The thermal energy regenerated is exchanged between the isobaric processes $C_1 E_2$ and E_3O.

Likewise the regenerative Stirling cycle, with his isothermal and isometric processes, may be easily represented.

Isothermal lines are here also isenthalpic lines and the graph may give help for the calculation of efficiencies.

Résumé—On donne ici quelques explications sur les propriétés du diagramme pression-entropie pour des liquides, la vapeur d'eau et les gaz parfaits.

La représentation sur ce graphique, non seulement des cycles thermodynamiques classiques, mais également des cycles modernes de réchauffage et de régénération montre qu'il peut être considéré comme un complément utile des diagrammes de Mollier.

Zusammenfassung—Die Eigenschaften des Druck-Entropie-Diagramms für Flüssigkeit, Dampf und ideale Gase werden erläutert. Ein solches Diagramm kann als brauchbare Ergänzung der Mollier-Diagramme angesehen werden, da es nicht nur die Kreisprozesse der klassischen Thermodynamik sondern auch moderne Prozesse mit Vorwärmung und Regeneration wiedergibt.

Аннотация—Дается объяснение некоторых особенностей p, s-диаграммы для жидкости, водяного пара и идеальных газов.

Построение этой диаграммы не только для классических термодинамических процессов, но также и для современных циклов с перегревом и регенерацией показывает, что её можно рассматривать как ценное дополнение к диаграмме Молье.